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Large Numbers Hypothesis. I. Classical Formalism[1]

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A self-consistent formulation of physics at the classical level embodying Dirac's large numbers hypothesis (LNH) is developed based on units covariance. A scalar "field" $\varphi(x)$ is introduced and some fundamental results are derived from the resultant equations. Some unusual properties of φ are noted such as the fact that φ cannot be the correspondence limit of a normal quantum scalar field.

1. INTRODUCTION

This paper is the first of a series which will develop a self-consistent formulation of physics embodying Dirac's (1937) large numbers hypothesis (LNH). Starting with the guiding principle of units covariance following the original suggestion of Dirac (1937), a measurable scalar "field" $\varphi(x)$ is introduced into standard physics in a self-consistent manner. The resultant field equations of classical physics are obtained and some fundamental results derived.

It must be emphasized at the outset that the mere existence of the large numbers discussed in Section 3 below does not require LNH to be valid. For example Carter (1974) has shown that if these numbers differed appreciably in magnitude from their current values then we (humans) would probably not exist to speculate about LNH. Further, Zel'dovich (1977) has noted that within modern quantum field theory spontaneous topology change can readily give rise to large numbers comparable in magnitude to those of LNH or even larger.

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This series of papers makes no claim for the necessity of LNH. The attitude adopted here is that since LNH specifically violates many of the cherished ideas of physics the formulation of a self-consistent theory encompassing LNH and still agreeing with all known observations can lead to a new understanding of the structure of standard physics. This in turn could enable one to examine the construction of quantum field theories from a radically different point of view. At each step of the way the existence of a self-consistent theory will allow specific observational tests to be developed.

Finally, I point out that this theory and its predictions are quite different from a superficially similar theory developed by Canuto, Adams, Hsieh, and Tsiang (1977a) and by Canuto, Hsieh, and Adams (1977b). The differences will become more readily apparent in subsequent papers dealing with the observation of phenomena mediated by electromagnetic radiation.

2. CONVENTIONS

This series of papers uses the $(+,-,-,-)$ metric, a comma denotes partial derivative, a semicolon denotes coordinate covariant derivative, an asterisk denotes units covariant derivative normalized to G units, a double vertical bar denotes units covariant derivative normalized to A units, the curvature tensor is defined by

$$A^{\mu}{}_{;\gamma\sigma} - A^{\mu}{}_{;\sigma\gamma} \equiv R^{\mu}{}_{\lambda\gamma\sigma} A^{\lambda} \tag{1}$$

and the Ricci tensor is

$$R_{\lambda\sigma} \equiv R^{\mu}{}_{\lambda\mu\sigma} \tag{2}$$

Greek indices run from 0 to 3 while Latin indices run from 1 to 3.

3. LARGE NUMBERS HYPOTHESIS

The large numbers hypothesis was first advanced by Dirac in 1937. Dirac noticed that while several pure numbers can be formed from the fundamental constants of physics, most are of the order of 137, 1836, etc. However, three such numbers are enormous. These are the ratio of the electric force to the gravitational force between an electron and a proton

$$\frac{e^2/r^2}{Gm_e m_p/r^2} = 0.22694(14) \times 10^{40} \tag{3}$$

the ratio of the "radius of the Universe" to the classical electron radius

$$\frac{c/H_0}{e^2/m_e c^2} = 6.57 \times 10^{40}\left(\frac{50}{H_0}\right) \tag{4}$$

and the "number of protons in the universe"

$$\frac{4}{3}\pi(c/H_0)^3(\rho_{m0}/m_p) = 0.0448 \times 10^{80}\left(\frac{\sigma_0}{0.03}\right)\left(\frac{50}{H_0}\right) \tag{5}$$

Here H_0 is the observed Hubble constant in km/sec/Mparsec, σ_0 is the observed density parameter and ρ_{m0} is the matter density today. The numbers in parentheses in (3) indicate the estimated one-standard-deviation uncertainty in the last two digits of the main number (Kelly et al., 1980).

Since H_0 decreases with the age of the universe the value of the second number is an accident due solely to our measuring H_0 today. The order-of-magnitude similarity between (4) and (3) led Dirac (1937) to suggest that perhaps this relation is not accidental. Dirac suggested that *every* large number with order of magnitude 10^{40} varies as the age of the universe. Since (3) contains only supposed constants this means that at least one of these "constants" must be variable. One is free to define fiducial units standards to be e and m_e. Hence the gravitational "constant" G must vary inversely with the age of the universe relative to these atomic units standards

$$G_A \sim t^{-1} \tag{6}$$

Similarly, the result (5) is interpreted as saying that the number of proton masses in the universe is increasing with the square of the age of the universe. Since the overwhelming majority of the number of different particles in the universe are protons this is interpreted as saying that the number of protons (matter quanta) in the universe is varying as

$$N_p \sim t^2 \tag{7}$$

Since 1937 two new numbers have been discovered. The first is the ratio of the "number of photons in the universe" to the "number of protons in the universe." This number became calculable after the discovery of the 2.7 K black-body radiation (Penzias and Wilson, 1965). If this radiation truly is cosmological then it accounts for the vast majority of photons in the universe. Taking the ratio of the "number density of photons" to the "number density of protons" gives

$$\frac{3.7aT_\gamma^3/k_B}{\rho_{m0}/m_p}=1.28\times10^{10}\left(\frac{0.03}{\sigma_0}\right)\left(\frac{50}{H_0}\right)^2\left(\frac{T_\gamma}{2.7}\right)^3 \tag{8}$$

Here k_B is Boltzmann's constant, a is the black body constant, and $T_\gamma=2.7$ K is the radiation black-body temperature. One can interpret (8) as saying that the ratio of the number of photons to the number of protons in the universe is increasing as the fourth root of the age of the universe

$$N_\gamma/N_p\sim t^{1/4} \tag{9}$$

or with (7)

$$N_\gamma\sim t^{9/4} \tag{10}$$

The second new large number is the ratio of the weak force between two weakly interacting masses (W bosons ?) and the gravitational force between these same two masses

$$\frac{g_v/R_w^4}{GM_w^2/R_w^2}=\frac{G_w(\hbar c)^3}{G(M_wR_w)^2}\simeq\frac{G_w\hbar c^5}{G4\pi^2}=44.052(27)\times10^{30} \tag{11}$$

Here g_v is the usual weak coupling constant of $V-A$ theory, G_w is the weak coupling constant of Weinberg-Salam (Kelly et al., 1980, p. 541), M_w is the weak particle mass, and R_w is the range of the weak interaction. A fascinating numerical coincidence arises when (11) is multiplied by the fine-structure constant α and then compared with the 3/4 power of (3)

$$\alpha\frac{G_w\hbar c^5}{4\pi^2G}=0.32146(20)\times10^{30} \tag{12a}$$

$$\left(\frac{e^2}{Gm_em_p}\right)^{3/4}=0.3280(18)\times10^{30} \tag{12b}$$

The two numbers agree to within 2%! One can interpret (12) as saying that the weak coupling "constant" G_w is varying inversely as the fourth root of the age of the universe relative to atomic units standards (Dicke, 1957)

$$G_w\sim t^{-1/4} \tag{13}$$

4. GUIDING PRINCIPLE

The point of this series of papers is to assume the validity of (6), (7), (10), and (13) and then to devise a complete dynamical formalism for physics which incorporates these phenomena. As stressed in the Introduction, it has been pointed out previously that the existence of the large numbers does not necessarily require LNH. That is not the point. Standard physics is automatically violated by each of (6), (7), (10), and (13). By constructing a complete, self-consistent dynamical formalism for physics which includes LNH one can devise rational experimental tests of LNH. This is not possible without such a theory. Further, and perhaps more important, by constructing such a formalism one should be able to learn much about standard physics, especially at the quantum level where all these phenomena must originate.

To begin development of such a theory first note that (6), (7), (10), and (13) each involve scalars in the tensorial sense (in fact, constant scalars). Thus the simplest possibility is to introduce a scalar field which will account for all the LNH phenomena, an approach utilized frequently in the past (Brans and Dicke, 1961; Deser, 1970; Dirac, 1973a; Freund, 1974). This scalar field will be called $\varphi(x)$. For practical purposes it can be thought of as the gravitational "constant" $G(x)$.

Notice that it is totally wrong to take a preexisting theory such as Newtonian mechanics, set $G = G(t)$ and then proceed to make predictions. This is because viable physical theories are tightly self-consistent logical structures. Making an arbitrary change in such a theory almost automatically guarantees a contradiction later within the internal structure of the theory. This is why all such attempts in the past have "proved" LNH to be wrong. In fact, all they really do is prove their own lack of self-consistency. I call such theories "naive Newtonian theories," and will often make comparisons between predictions of this self-consistent theory and predictions of the naive Newtonian theories.

One seeks a self-consistent method to introduce φ. The guiding principle has been discussed many times by Dirac (1937; 1973a,b; 1974). Notice that the large numbers (3), (4), (5), and (11) are ratios between microscopic or atomic quantities (e, h, m_e, etc.) and macroscopic or gravitational quantities ($G, \rho_m = M/V$). Since one is always free to choose fiducial units standards, one can choose to define masses in terms of m_e, lengths in terms of $e^2/m_e c^2$, etc. The system of units standards based on such atomic quantities will be called A units. Alternatively, one could choose the mass of the sun, $M_\odot$, to be the fiducial mass and $GM_\odot/c^2$ to be the fiducial length. The system of units standards based on such macroscopic quantities will be called G units.

Clearly e, m_e, h, etc. are all constant in A units by definition. Clearly G and $M_\odot$ are constant in G units by definition. Standard physics assumes that ratios of A units to G units are always constant. Dirac (1937; 1973a, b; 1974) pointed out that if this assumption is dropped then LNH could result. Consequently, assume that Nature admits two natural units standards. In G units all classical (nonquantum) quantities have their usual form. This means that in G units macroscopic masses and charges and the gravitational constant are all constant. Further, in G units classical electromagnetic theory and classical gravitation are valid

$$F_{\lambda\sigma;\mu} + F_{\sigma\mu;\lambda} + F_{\mu\lambda;\sigma} = 0 \tag{14a}$$

$$F^{\lambda\sigma}{}_{;\sigma} = 4\pi\sigma\mu^{\lambda} \tag{14b}$$

$$G_{\lambda\sigma} + \Lambda g_{\lambda\sigma} = -8\pi G T_{\lambda\sigma} \tag{15}$$

where σ is the proper charge density and Λ is the cosmological constant. On the other hand, in A units all atomic quantities have their usual form, i.e., h, m_e, e, m_p, transition frequencies of atoms, etc., are all constant. One must fix up the mathematics to respect this principle. Hence one must create units covariance and give it measurable dynamic significance.

5. UNITS COVARIANT FIELD EQUATIONS

A. General Formulation. The reader unfamiliar with the units covariant formalism should refer to the Appendix. There a units covariant formalism based on the two independent quantities of physics, mass and length (time), is developed. Notice that one is free to define length in terms of light travel time ($c \equiv 1$) so that all physics *does* reduce to just the two independent quantities mass and length.

Since we know the form of the field equations (14) and (15) in G units one normalizes the scalar gauge field $\beta(x)$ so that when $\beta \equiv 1$ the units covariant equations reduce to the G units equations (14) and (15). This means that the fiducial units are G units, i.e.,

$$d\tau = \beta^{-1}(x)\, d\tau_G \tag{16}$$

is the required expression for time in arbitrary units in terms of time in G units.

One could similarly introduce a second gauge variable $\chi(x)$ to relate an arbitrary mass unit to G units as

$$dM = \chi^{-1}(x)\, dM_G \tag{17}$$

However, simplicity together with the form of LNH suggest that the effects of LNH be mediated by one scalar field, not two. Since the only reason for introducing units covariance is to indicate how the scalar field $\varphi(x)$ should be inserted into the standard physics field equations, one can simplify the general units covariance by taking

$$\chi(x) \equiv [\beta(x)]^{1-g} \tag{18}$$

where g is some constant. Hence

$$dM = [\beta(x)]^{g-1}\, dM_G \tag{19}$$

is the required expression for mass in arbitrary units in terms of mass in G units.

As noted in the Appendix every physical quantity must possess both an explicit tensorial nature and an explicit *power* (Dirac, 1973a). The power of any quantity A is determined from the units of A as

$$\Pi(A) = \alpha \equiv a + b(1-g) \tag{20a}$$

if

$$[A] = [T]^a [M]^b \tag{20b}$$

where $[A]$ denotes the units of A. Hence the algebra of powers reduces to the algebra of dimensional analysis. For example, the power of G is found to be

$$[G] = [L][M]^{-1} \Rightarrow \Pi(G) = 1-(1-g) = g \tag{21}$$

As noted in the Appendix a units covariant derivative (asterisk derivative) (Dirac, 1973a) can be defined. It is related to the standard coordinate covariant derivative (semicolon derivative) through the normalization of $\beta(x)$

$$\beta(x) \equiv 1 \Rightarrow * = ; \tag{22}$$

This allows one to use a semicolon-to-star rule to change any equation in coordinate covariant form into an equivalent equation in units covariant form. For example, the units covariant electromagnetism equations (14) become

$$F_{\lambda\sigma*\mu} + F_{\sigma\mu*\lambda} + F_{\mu\lambda*\sigma} = 0 \tag{23a}$$

$$F^{\lambda\alpha}{}_{*\alpha} = 4\pi\sigma\mu^{\lambda} \tag{23b}$$

However, the units covariant gravitational field equations are more subtle. From (15) one must replace the Einstein tensor $G_{\lambda\sigma}$ with its units covariant equivalent form (A.15) to get (Dirac, 1973a)

$$*G_{\lambda\alpha}+\Lambda g_{\lambda\sigma}=-8\pi GT_{\lambda\sigma} \tag{24}$$

Notice that in (23) and (24) all quantities are assumed to be in units covariant form. For example, from (21) one has

$$G=\beta^{-g}G_G \tag{25}$$

where G_G is constant by definition. Similarly, from (A.18), (A.21), and (24)

$$\Pi(\Lambda g_{\lambda\sigma})=\Pi(\Lambda)+2=\Pi(*G_{\lambda\sigma})=0 \tag{26}$$

$$\Lambda=\beta^2\Lambda_G \tag{27}$$

where Λ_G is constant since (15) is valid in G units. Also, from (24), (A.18), (A.21), and (21) one finds

$$\Pi(T_{\sigma\lambda})=\Pi(*G_{\sigma\lambda})-\Pi(G)=-g \tag{28}$$

$$\Pi(T_\sigma^{\lambda})=\Pi(T_{\sigma\lambda})+\Pi(g^{\sigma\lambda})=-g-2 \tag{29}$$

$$\Pi(T^{\sigma\lambda})=\Pi(T_\sigma^{\lambda})+\Pi(g^{\sigma\lambda})=-g-4 \tag{30}$$

Since

$$T_\sigma^{\lambda}=\frac{1}{4\pi}\left(F_\sigma{}^{\rho}F_\rho^{\lambda}-\tfrac{1}{4}\delta_\sigma{}^{\rho}F_\alpha{}^{\rho}F_\rho{}^{\alpha}\right) \tag{31}$$

for electromagnetism one deduces from (31), (29), and (A.18) that

$$\Pi(F_\sigma^{\lambda})=\tfrac{1}{2}\Pi(T_\sigma^{\lambda})=-1-g/2 \tag{32}$$

$$\Pi(F_{\sigma\lambda})=\Pi(F_\sigma^{\lambda})+\Pi(g_{\sigma\lambda})=1-g/2 \tag{33}$$

$$\Pi(F^{\sigma\lambda})=\Pi(F_\sigma^{\lambda})+\Pi(g^{\sigma\lambda})=-3-g/2 \tag{34}$$

Hence (A.6) and (A.29) combined with (23) give

$$\left(\beta^{1-g/2}F_{\lambda\sigma}\right)_{;\mu}+\left(\beta^{1-g/2}F_{\sigma\mu}\right)_{;\lambda}+\left(\beta^{1-g/2}F_{\mu\lambda}\right)_{;\sigma}=0 \tag{35a}$$

$$\left(\beta^{1-g/2}F^{\lambda\alpha}\right)_{;\alpha}=4\pi\beta^{1-g/2}\sigma u^{\lambda} \tag{35b}$$

Just as in standard physics (35a) implies the existence of A_λ such that

$$\beta^{1-g/2}F_{\lambda\sigma}=\left(\beta^{1-g/2}A_\lambda\right)_{;\sigma}-\left(\beta^{1-g/2}A_\sigma\right)_{;\lambda} \tag{36a}$$

From (36a) the units of A_λ are the same as those of $F_{\lambda\sigma}$ so

$$\Pi(A_\lambda)=\Pi(F_{\lambda\sigma})=1-g/2 \tag{37}$$

which means that (36a) can be written as

$$F_{\lambda\sigma}=A_{\lambda*\sigma}-A_{\sigma*\lambda} \tag{36b}$$

Equations (35) are the electromagnetic field equations written in explicit coordinate covariant form with arbitrary units (variable mass standards, rubber length standards, "poorly" built standard clocks, etc.).

B. Physics in *A* Units. Now consider an A clock, e.g., the period of vibration of an ammonia molecule. The readings of the A clock can be compared with the readings of a G clock, e.g., the orbital period of a planet about the Sun. Define

$$\varphi\equiv d\tau_G/d\tau_A \tag{38}$$

where $d\tau_G$ is the time interval between two events measured by the G clock and $d\tau_A$ is the time interval between two events measured by the A clock. From (16) this means that

$$\beta=1\leftrightarrow G\text{ units} \tag{39a}$$

$$\beta=\varphi\leftrightarrow A\text{ units} \tag{39b}$$

Since both G clocks and A clocks do exist this means that φ is a measurable quantity. Thus all of the units covariant equations of classical physics can be written immediately in terms of A units by making the gauge choice $\beta=\varphi$. $\beta(x)$ is a gauge variable and, like all gauge variables, is inherently unmeasurable. Fixing the gauge by choosing $\beta=1$ means that physics is described in terms of G units. Fixing the gauge by choosing $\beta=\varphi$ means that physics is described in terms of A units. ALL OF THIS IS TRUE IN STANDARD PHYSICS!

In standard physics φ is constant. Hence there is at most a constant factor difference between G units and A units. Dirac's suggestion concerning the existence of two natural units standards is equivalent to the assertion that φ is not constant (Dirac, 1973a)

$$\varphi=\varphi(x) \tag{40}$$

From (35) with $\beta=\varphi$ it is clear that this immediately changes standard physics. Further, use of (A.15), (25), and (27) in (24) with $\beta=\varphi$ gives

$$G_{\lambda\sigma}+2\frac{\varphi;\lambda\sigma}{\varphi}-4\frac{\varphi,\lambda\varphi,\sigma}{\varphi^2}-g_{\lambda\sigma}\left(2\frac{\Box\varphi}{\varphi}-\frac{\varphi,\rho\varphi,\mu}{\varphi^2}g^{\rho\mu}\right)$$
$$=-\Lambda_G\varphi^2-8\pi G_G\varphi^{-g}T_{\lambda\sigma} \tag{41}$$

for the gravitational field equation in A units which is radically different from the standard form (15).

Many people have obtained the left-hand side of (41) by insisting that gravity be units covariant or scale covariant. However, most people did not apply the same requirement to the source terms on the right-hand side. It is imperative that the same requirements of units covariance or scale covariance be applied to the right-hand side of (41) as well as the left-hand side. The reason is that φ cannot be a normal classical scalar field, i.e., the correspondence limit of the usual quantum scalar field. If φ were such a field then φ would affect the kinematics of quantum systems. But this is precisely what φ cannot do or the entire concept of A units being dynamically different from G units is lost. φ must be such that it affects the dynamics of quantum systems but not the kinematics of quantum systems. φ may cause transitions among quantum states but the states themselves must be unaffected by φ.

So far units covariance has been utilized to introduce φ into physics in a self-consistent manner. It has also been utilized to determine the variation of "constants" such as G or Λ when measured in A units. Now units covariance will be used to determine the variation of atomic quantities such as e, m_e, h, etc., when measured in terms of G units.

m_e is a mass and so has power $1-g$ by (19). In atomic units m_e is constant. Hence any atomic mass is expressible as

$$m=m_A(\beta/\varphi)^{g-1} \tag{42}$$

where m is the mass in arbitrary units and m_A is the constant mass in A units. Since h/mc is a length

$$\Pi(h)=\Pi(L)+\Pi(m)=1+1-g=2-g \tag{43a}$$

so

$$h = h_A(\beta/\varphi)^{g-2} \tag{43b}$$

where h is Planck's "constant" in arbitrary units and h_A is the constant value of h in A units. Since e^2/mc^2 is also a length

$$\Pi(e) = \tfrac{1}{2}\Pi(e^2) = \tfrac{1}{2}\Pi(h) = 1 - g/2 \tag{44a}$$

so

$$e = e_A(\beta/\varphi)^{-1+g/2} \tag{44b}$$

In A units m, h, and e are constant as expected while in G units they are given by (42), (43b), and (44b) with $\beta = 1$.

C. Units Covariant Variational Principle (Adams, 1979). In this section I give a units covariant variational principle for gravity and electromagnetism coupled to a charged perfect fluid. The energy tensor for the perfect fluid is

$$T^{\lambda\sigma} = (\rho + p)u^\lambda u^\sigma - p g^{\lambda\sigma} \tag{45}$$

where p is the isotropic fluid pressure and ρ is the proper energy density of the fluid. Notice that in the classical units covariant equations (23) and (24) φ nowhere appears. This will always be true since φ is derived by comparing atomic quantities with classical quantities and classical physics contains no atomic quantities by definition. Hence φ will nowhere appear in the variational integral.

Write

$$I = I_g + I_{em} + I_q + I_m \tag{46a}$$

$$I_g \equiv (16\pi G_G)^{-1} \int ({}^*R\beta^2 - 2\Lambda_G \beta^4)\surd d^4x \tag{46b}$$

$$I_{em} \equiv (16\pi)^{-1} \int \beta^{2f} g^{\mu\alpha} g^{\sigma\lambda} (F_{\alpha\lambda} - 2A_{\alpha * \lambda} + 2A_{\lambda * \alpha}) F_{\mu\sigma} \surd d^4x \tag{46c}$$

$$I_q \equiv -\int \beta^{s+f+3} \sigma u^\alpha A_\alpha \surd d^4x \tag{46d}$$

$$I_m \equiv -\int \rho \beta^{m+4} \surd d^4x \tag{46e}$$

$$\Pi(\rho) \equiv m, \qquad \Pi(F_{\mu\sigma}) = \Pi(A_\lambda) \equiv f \tag{46f}$$

$$\surd \equiv \det(g_{\lambda\sigma}), \qquad \Pi(\sigma) \equiv s \tag{46g}$$

with g, m, f, and s constants to be determined. The power of the four-velocity u^α is found to be

$$\Pi(u^\alpha) = \Pi(dx^\alpha) - \Pi(d\tau) = -1 \tag{47}$$

Notice that the power of I is zero. Hence I is automatically invariant under an arbitrary units (gauge) transformation. Variation of I_g, I_{em}, I_q, and I_m gives

$$\delta I_g = (16\pi G_G)^{-1} \int \Big\{ \big(\beta^{2*}G_{\mu\sigma} + \beta^2 \Lambda_G g_{\mu\sigma}\big) \delta g^{\mu\sigma} + \big(2\beta^* R - 8\beta^3 \Lambda_G\big) \delta\beta \Big\} \surd d^4x \tag{48a}$$

$$\begin{aligned} \delta I_{em} = (16\pi)^{-1} \int \Big\{ & 2\beta^{2f} \Big[F_\mu{}^\lambda \big(2A_{\lambda*\sigma} - 2A_{\sigma*\lambda} - F_{\lambda\sigma}\big) \\ & + \tfrac{1}{4} g_{\mu\sigma} F^{\alpha\lambda} \big(2A_{\alpha*\lambda} - 2A_{\lambda*\alpha} - F_{\alpha\lambda}\big) \Big] \delta g^{\mu\sigma} \\ & + 4\beta^{2f} F^{\alpha\lambda}{}_{*\lambda} \delta A_\alpha + 2\beta^{2f} g^{\alpha\mu} g^{\lambda\sigma} \big(F_{\alpha\lambda} - A_{\alpha*\lambda} + A_{\lambda*\alpha}\big) \delta F_{\mu\sigma} \\ & + 2f\beta^{2f-1} \Big[2A_\alpha F^\lambda{}_{*\lambda} + F^{\alpha\lambda} \big(F_{\alpha\lambda} - A_{\alpha*\lambda} + A_{\lambda*\alpha}\big) \Big] \delta\beta \Big\} \surd d^4x \end{aligned} \tag{48b}$$

$$\delta I_q = \int \Big\{ \beta^{f+s+3} \sigma u^\alpha \big(A_{\lambda*\alpha} - A_{\alpha*\lambda}\big) \delta x^\lambda - \beta^{f+s+3} \sigma u^\alpha \delta A_\alpha - f\beta^{f+s+2} \sigma A_\alpha u^\alpha \delta\beta \Big\} \surd d^4x \tag{48c}$$

$$\begin{aligned} \delta I_m = \int \Big\{ & \beta^{m+4} \Big[(\rho + p) u_{\lambda*\alpha} u^\alpha - p_{*\alpha} \big(\delta^\alpha{}_\lambda - u^\alpha u_\lambda\big) \Big] \delta x^\lambda \\ & + \tfrac{1}{2} \beta^{m+4} \Big[(\rho + p) u_\mu u_\sigma - p g_{\mu\sigma} \Big] \delta g^{\mu\sigma} - \beta^{m+3} (\rho - 3p) \delta\beta \Big\} \surd d^4x \end{aligned} \tag{48d}$$

Writing $\delta I = \delta I_g + \delta I_{em} + \delta I_q + \delta I_m$ and setting the coefficients of the independent variations equal to zero gives the field equations. From $\delta F_{\mu\sigma}$

$$F_{\mu\sigma} = A_{\mu*\sigma} - A_{\sigma*\mu} \tag{49}$$

From δA_μ

$$F^{\mu\lambda}{}_{*\lambda} = 4\pi\beta^{s+3-f}\sigma u^\mu \tag{50a}$$

$$s = f - 3 \tag{50b}$$

where (50b) stems from the usual condition that the field equations have no explicit dependence on the gauge variable β when written in units covariant form. From $\delta g^{\mu\sigma}$

$$\begin{aligned} {}^*G_{\mu\sigma} + \Lambda_G \beta^2 g_{\mu\sigma} = -8\pi G_G \big\{ \beta^{m+2}\big[(\rho + p)u_\mu u_\sigma - p g_{\mu\sigma}\big] \\ + \beta^{2f-2}\big[F_\mu^{\ \lambda} F_{\lambda\sigma} - \tfrac{1}{4} g_{\mu\sigma} F_\alpha^{\ \lambda} F_\lambda^{\ \alpha}\big]/4\pi \big\} \end{aligned} \tag{51}$$

The equivalence principle [Eötvös–Dicke–Braginsky experiment (Misner, et al., 1973)] requires

$$m + 2 = 2f - 2 \tag{52}$$

since matter and electromagnetism must couple to gravity in the same way. Thus

$$^*G_{\mu\sigma} + \Lambda g_{\mu\sigma} = -8\pi G T_{\mu\sigma} \tag{53}$$

requires

$$\Lambda = \Lambda_G \beta^2, \qquad G = G_G \beta^{2f-2} \equiv G_G \beta^{-g} \tag{54}$$

or

$$f = 1 - g/2 \tag{55}$$

so (50b) and (52) give

$$s = -2 - g/2, \qquad m = -2 - g \tag{56}$$

consistent with what was learned previously. From δx^λ

$$(\rho + p) u^\mu{}_{*\lambda} u^\lambda = p_{*\lambda}(g^{\mu\lambda} - u^\mu u^\lambda) + \sigma u^\lambda F_\lambda^{\ \mu} \tag{57}$$

where (50b), (55), and (56) were used. Equation (57) is recognized to be the units covariant Euler equation for a charged fluid. Finally, the $\delta\beta$ variation gives

$$^*R - 4\Lambda = 8\pi G T^\lambda{}_\lambda \tag{58}$$

where (49), (50), (55), and (56) were used. Equation (58) is not an independent equation for β since it is just the trace of (53). This is consistent with the well-known condition that a gauge-covariant theory does not admit field equations for the gauge functions.

D. Measurability of φ. I emphasize that φ is directly measurable whereas the gauge function β is not. All measurements are in fact ratios of similar quantities. An unknown mass is balanced by a certain number of standard masses. An unknown length is a certain number of standard lengths. Hence the units of mass, length, time, etc. (the power of the quantity being measured) divide out (the measurement has zero power since it is a pure number). Consequently, such a ratio will never contain β, i.e., β is inherently unmeasurable.

However, such a ratio can contain φ. If a classical mass is measured in terms of atomic mass standards φ will appear in the result. This is precisely what is meant by using A units or by the atomic gauge $\beta = \varphi$. Hence to measure φ one seeks a necessarily A unit measurement of a necessarily G unit object.

Precisely this situation arises in gravitationally induced quantum interference experiments (Colella et al., 1975). What is measured is the quantum-mechanical phase shift of atomic particles (neutrons) caused by their interaction with a gravitational field. In effect, one measures G in A units. Ideally, one could count out 10^6 moles of ^{56}Fe atoms and form them into a sphere in space. One could then determine the gravitational constant

$$G(t_i) = G_0(\varphi_0/\varphi(t_i))^g \tag{59}$$

at different times t_i separated by, say, 10^6 years. Each time the experiment is performed one could set it up so that 10^6 moles of ^{56}Fe are used and the neutron beam paths are the same (this eliminates any complications due to matter replication, etc.). Then

$$\varphi(t_i)/\varphi_0 = [G_0/G(t_i)]^{1/g} \tag{60}$$

which determines φ to within a constant factor if g has been determined independently.

6. LNH AND THE FORM OF φ

So far there is no way to determine φ theoretically. Since φ is not (and cannot be) contained in the variational integral (46) there is no possibility of obtaining a field equation for φ from classical physics. However, the form of LNH given by (6) and (7) can be used to estimate a form for $\varphi(x)$. Since

$$G_A = \varphi^{-g} G_G = G_0(\varphi/\varphi_0)^{-g} \tag{61}$$

from (25) one can use (6) to find

$$(\varphi/\varphi_0)^g = t/t_0 \tag{62}$$

where t is (comoving) cosmic time in A units and t_0 is the value of t today.

In order to determine g one uses (7) together with (19) and (42). The number N_m of atomic masses m in a classical mass M is

$$N_m = M/m = M_G \beta^{g-1}/m_A(\beta/\varphi)^{g-1} = N_{m0}(\varphi/\varphi_0)^{g-1} \tag{63}$$

One could use (7) directly to find g. However, an additional factor enters here. When Dirac proposed (7) he pointed out that LNH does not indicate *where* matter is being created. There are three possibilities (Dirac, 1973b): (i) Matter is replicating (multiplicative creation); (ii) matter is being created uniformly throughout all space (additive creation); (iii) matter is being created at certain preferred points in the universe, e.g., galactic nuclei (local creation). If additive creation holds then since the universe is mostly empty the amount of matter being created in the vicinity of matter already present, in say, 10^{10} years is almost zero everywhere. Hence (63) would be almost constant, i.e., $g \cong +1$. If local creation holds then the amount of matter being created in the vicinity of matter already present in 10^{10} years is zero almost everywhere. Again (63) would be constant, i.e., $g = +1$. However, if multiplicative creation holds then (63) should vary like (7). Combining with (62) gives $g = -1$. In summary we have the following:

Additive or local creation:

$$g = +1, \qquad \varphi/\varphi_0 \cong t/t_0 \tag{64}$$

Multiplicative creation:

$$g = -1, \qquad \varphi/\varphi_0 \cong t_0/t \tag{65}$$

I will show in Paper II of this series that there are compelling reasons to believe that $g = -1$ (multiplicative creation) if this formalism is valid at all.

7. NEUTRAL PERFECT FLUIDS

In this section I discuss properties of neutral perfect fluids in the units covariant formalism. The energy tensor for a perfect fluid is given by (45). $\Pi(\rho)=\Pi(p)=-2-g$ from (56) consistent with ρ being a mass per unit volume. In standard physics one deduces the equations of motion for a perfect fluid from $T^{\lambda\alpha}{}_{;\alpha}\equiv 0$ which follows from Einstein's equation (15). In this formalism one uses (24), (A.16), (A.22), and (A.23) to deduce the units covariant analog

$$T^{\lambda\alpha}{}_{*\alpha}\equiv 0 \tag{66}$$

Substitution of (45) into (66) yields

$$(\rho+p)u_{\rho*\alpha}u^{\alpha}=p_{*\rho}-[(\rho+p)u^{\alpha}]_{*\alpha}u_{\rho} \tag{67}$$

From $u_{\rho}u^{\rho}=1$ one has

$$u^{\rho}u_{\rho*\alpha}=0 \tag{68}$$

as in standard physics, so contraction of (67) with u^{ρ} gives

$$[(\rho+p)u^{\alpha}]_{*\alpha}=p_{*\alpha}u^{\alpha} \tag{69}$$

and (67) becomes

$$(\rho+p)u^{\lambda}{}_{*\alpha}u^{\alpha}=p_{*\alpha}(g^{\lambda\alpha}-u^{\lambda}u^{\alpha}) \tag{70}$$

Equations (69) and (70) are the generalized energy and Euler equation in units covariant form. In the limit of negligibly small pressure forces these become

$$(\rho u^{\alpha})_{*\alpha}=0 \tag{71}$$

$$u^{\lambda}{}_{*\alpha}u^{\alpha}=0 \tag{72}$$

Use of (A.8) and (A.27) gives

$$(\beta^{1-g}\rho u^{\alpha})_{;\alpha}=0 \tag{73}$$

$$u^{\lambda}{}_{;\alpha}u^{\alpha}=\frac{\beta_{,\alpha}}{\beta}(g^{\lambda\alpha}-u^{\lambda}u^{\alpha}) \tag{74}$$

in explicit covariant form in arbitrary units.

Equation (73) leads back to the matter equation (19) since it implies

$$\int_{t=\text{const}} \beta^{1-g}\rho u^0\, dV = \text{const} \tag{75}$$

and for $\beta = \beta(t)$

$$M = M_0(\beta/\beta_0)^{g-1} \tag{76}$$

Since $M = N_m$ where m is an atomic mass (42) and (76) give

$$N_m = N_{m0}(\varphi/\varphi_0)^{g-1} \cong N_{m0}(t/t_0)^{1-1/g} \tag{77}$$

from (62). Hence the number of atomic particles in a classical mass increases or decreases as $g^{-1}<1$ or $g^{-1}>1$, respectively. This shows how particle replication is built into the theory [which is of course, based on (19) and (42)].

Equation (74) is called the in-geodesic equation by Dirac (1973a). It is the equation of motion for free test particles in arbitrary units. In G units it reduces to the usual geodesic equation as expected. In A units (74) shows that in a frame where $\varphi = \varphi(t)$ (which always exists in an isotropic, homogeneous universe) in the Minkowski space-time limit ($g_{\lambda\sigma} \cong \eta_{\lambda\sigma}$) the physical velocity of a particle measured in A units satisfies

$$v = \gamma_0 v_0 \varphi_0 \left(\varphi^2 + \gamma_0^{\,2} v_0^{\,2} \varphi_0^{\,2}\right)^{-1/2} \tag{78a}$$

$$\gamma^2 = 1 + \gamma_0^{\,2} v_0^{\,2} (\varphi_0/\varphi)^2 \cong 1 + \gamma_0^{\,2} v_0^{\,2} (t_0/t)^{2/g} \tag{78b}$$

where $\gamma_0 = \gamma(t_0)$, etc., and (62) was used. Hence a free particle whose velocity is measured in A units should spontaneously slow down or speed up according as $g>0$ or $g<0$, respectively. The acceleration of such a particle is found to be

$$\mathbf{a} = -\frac{\dot{\varphi}}{\varphi}\frac{\mathbf{v}}{\gamma^2} \cong \frac{1}{(-g)}\frac{\mathbf{v}}{t\gamma^2} \tag{79}$$

8. CLASSICAL CHARGED MATTER

The field equations for classical charged matter in arbitrary units are the units covariant Maxwell equations (23) together with (57). In the limit of negligible pressure (57) becomes

$$\rho u^{\lambda}{}_{*\alpha} u^{\alpha} = \sigma u^{\alpha} F_{\alpha}{}^{\lambda} \tag{80a}$$

or

$$u^{\lambda}{}_{;\alpha} u^{\alpha} = \frac{\beta_{,\alpha}}{\beta}\left(g^{\lambda\alpha} - u^{\lambda} u^{\alpha}\right) + \frac{\sigma}{\rho} u_{\alpha} F^{\alpha\lambda} \tag{80b}$$

Clearly (80a) is the units covariant generalization of the Lorentz force equation, while (80b) illustrates how the Lorentz force term adds on to the free particle in-geodesic equation. From the form of (80b) in A units ($\beta = \varphi$) it is clear that the first term on the right acts as a sort of generalized cosmic force. In fact it originates in the microstructure of space-time as will be shown in a future paper.

Equation (23b) implies

$$(\sigma u^{\alpha})_{*\alpha} = 0 \tag{81}$$

due to the antisymmetry of $F^{\lambda\alpha}$ together with (A.27) and (A.29). From (56) and (A.27) this gives

$$\left(\beta^{1-g/2} \sigma u^{\alpha}\right)_{;\alpha} = 0 \tag{82}$$

as the law of charge "conservation". Equation (82) implies

$$\int_{t=\text{const}} \beta^{1-g/2} \sigma u^0 \, dV = \text{const} \tag{83}$$

and for $\beta = \beta(t)$

$$Q = Q_0 (\beta/\beta_0)^{-1+g/2} \tag{84}$$

where Q is classical charge. The power of Q is consistent with (44a) as required. Since $Q = e\Delta N$ where e is a proton charge and ΔN is the net charge number, (84) and (44b) give

$$\Delta N = \Delta N_0 (\varphi/\varphi_0)^{-1+g/2} \cong \Delta N_0 (t/t_0)^{1/2-1/g} \tag{85}$$

from (62). Hence the *net* number of atomic charges in a classical charge increases or decreases as $g^{-1} < 1/2$ or $g^{-1} > 1/2$, respectively. Comparison with (77) shows that this is an unexpected result because it predicts that net charge creation and particle creation do not occur at the same rate.

However, from (77), (85), and (62) one finds

$$\frac{Q}{M}=\frac{Q_0}{M_0}(\varphi/\varphi_0)^{-g/2}\cong\frac{Q_0}{M_0}(t_0/t)^{1/2} \tag{86}$$

so that in A units a classical charge-to-mass ratio continuously decreases. Consequently, this theory does not admit perfect replication of matter since otherwise the charge-to-mass ratio would be constant. Hence, the presence of one particle type must affect the creation rate of other particle types. Note that the suggestion that particle replication rates depend on environment is not so *ad hoc* as it seems, since the particle replication rate of a system of completely degenerate fermions must be zero in order that the Pauli principle not be violated.

9. DISCUSSION

I have presented details of a formalism which sensibly allows one to incorporate LNH into physics at the classical level. By examining progressively less classical systems one expects to be able to trace the effects and origin of φ down to the quantum level. $\varphi(x)$ is a scalar "field" for which no field equation can (or should) exist at the classical level since φ is a manifestation of an effect operating solely at the quantum level. φ cannot be merely the correspondence limit of a standard quantum field for otherwise it would affect quantum kinematics. This it must not do in order that the dynamical significance of and differences between A units and G units be preserved. φ can be allowed to affect only the standard quantum dynamics, i.e., φ causes transitions among quantum states but does not affect the states themselves.

The sole purpose of the units covariance approach is to allow a rational, self-consistent introduction of a new scalar field φ into physics. Since laboratory measurements are made using A units almost exclusively, the units covariant equations should be evaluated with $\beta=\varphi$. This leads to the conclusion that G and Λ vary and that free particles follow in-geodesics. In general one finds that the existence of φ means that systems do not conserve energy in A units. It is as though all matter in our universe is weakly coupled to a large energy source or sink (depending on the value of g). This point will be developed in Paper III on thermodynamics.

Almost all previous papers attempting to incorporate LNH into physics fail because they treat φ as a normal scalar field (for example, Bekenstein, 1977, and papers cited there). Hence, these previous papers break units covariance (or scale covariance as some call it) in order to obtain field equations for φ. (For example, this theory is a singular $\omega = -3/2$ case of Brans–Dicke.) As soon as this is done the dynamical differences between A units and G units disappear.

Notable exceptions to this are the original works by Canuto, Adams, Hsieh, and Tsiang (1977a) and by Canuto, Hsieh, and Adams (1977b) which has been applied by Canuto and co-workers (Canuto and Hsieh, 1979; Canuto et al., 1979; Canuto and Owen, 1979). While these formulations are superficially similar to this theory there are major differences in results. This theory uses units covariance (called scale covariance in the above papers) as a means to self-consistently insert φ. At no time will explicit units covariance ever need to be broken. Hence equations such as (42), (43b), and (44b) are obtained which do not exist in the above papers. The biggest difference arises in the treatment of radiation. Because of its importance this is discussed separately in Paper II.

Finally, one vital point must be repeated. It is fruitless to try to show that some particular theory disagrees with observation by using a *different* theory as an intermediary. This could be similar to "disproving" standard thermodynamics through use of the theory of caloric.

Two examples from the recent literature will suffice to illustrate this point. In the first example (Barrow, 1978) an attempt was made to rule out the scale covariant theory of Canuto et al. (1977a) and other theories by showing that cosmological nucleosynthesis in the early universe severely restricts the possible temporal variation of G. This was a worthy effort except that standard physics [with $G(t)$] was used instead of the theory being tested. In fact, the theory in question makes no claim to being sufficiently developed to be able to calculate cosmological nucleosynthesis in the early universe.

In the second example (Mansfield and Malin, 1980) the scale covariant energy equation similar to (69) was applied to degenerate stars and severe difficulties in understanding certain observations were obtained. However, a degenerate star, or any degenerate quantum system, is precisely the kind of system for which (69) would be expected to fail. Equation (69) stems from LNH in the form (19) and (25) (with $\beta = \varphi$). While (25) may still hold in degenerate quantum systems, (19) most assuredly will not. Since most matter is fermionic a degenerate system of fermions cannot replicate due to the Pauli principle (except possibly at the top of the Fermi sea). How this would modify equation (19) is anybody's guess. Consequently, any conclusions drawn from such an application of (19) are almost guaranteed to be false.

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APPENDIX: UNITS COVARIANCE

As stressed in the body of this paper units covariance has no physical content of and by itself. Units covariance is an explicit statement of the fact that physics is independent of the particular set of units standards used. In this sense it is completely analogous to coordinate covariance which is an explicit statement of the fact that physics is independent of the particular set of coordinates used. Coordinate covariance can be explicitly introduced into special relativity through the "comma-to-semicolon rule," which in no way affects the physical content of the theory. Similarly, units covariance can be explicitly introduced into general (or special) relativity through the "semicolon-to-star rule" which in no way affects the physical content of the theory.

All physics is ultimately based on $F = ma$. Hence there are precisely three independent units in physics, mass, length, and time. Defining units of length in terms of light travel time (setting $c \equiv 1$) reduces the number of independent units to two, mass and time (length). Introduce a gauge variable $\beta(x)$ to keep track of variable time standards. Then an arbitrary time standard is related to a fiducial time standard by

$$d\tau \equiv \beta^{-1}(x)\, d\tau_F \tag{A.1}$$

Similarly, one can introduce a second gauge variable $\chi(x)$ to keep track of variable mass standards. Then an arbitrary mass standard is related to a fiducial mass standard by

$$dM \equiv \chi^{-1}(x)\, dM_F \tag{A.2}$$

To determine the units transformation properties of any quantity A simply determine the units of A as

$$[A] = [T]^a [M]^b \tag{A.3}$$

Then in a system of arbitrary units standards one has

$$A=\beta^{-a}\chi^{-b}A_F \tag{A.4}$$

The quantity A is said to have β power a and χ power b and one writes

$$a=\Pi_\beta(A), \qquad b=\Pi_\chi(A) \tag{A.5}$$

Every physical quantity must possess both an explicit tensorial nature and explicit β and χ powers. The algebra of powers reduces to the algebra of dimensional analysis.

Just as one must introduce a semicolon derivative to preserve coordinate covariance, so too must one introduce a star derivative to preserve units covariance. Note that any coordinate has power zero. This is because coordinates are just markers in space-time and have no units associated with them. The unit of time (length) normally associated with coordinates is actually carried by the metric tensor $g_{\mu\sigma}$. Consequently, the units covariant derivative of a quantity must have the same power as the original quantity.

Define the symmetric affinity

$$^*\Gamma^\mu_{\lambda\sigma}\equiv\Gamma^\mu_{\lambda\sigma}+\delta^\mu_\lambda\frac{\beta_{,\sigma}}{\beta}+\delta^\mu_\sigma\frac{\beta_{,\lambda}}{\beta}-g_{\beta\sigma}g^{\mu\rho}\frac{\beta_{,\rho}}{\beta} \tag{A.6}$$

where $\Gamma^\mu_{\lambda\sigma}$ is the Christoffel symbol of Riemannian geometry. Then the units covariant derivative is the same as the coordinate covariant derivative of Riemannian geometry with $^*\Gamma^\mu_{\lambda\sigma}$ replacing $\Gamma^\mu_{\lambda\sigma}$ except for two additional terms involving power. Thus

$$A_{*\sigma}=A_{,\sigma}+aA\frac{\beta_{,\sigma}}{\beta}+bA\frac{\chi_{,\sigma}}{\chi} \tag{A.7}$$

$$A^\mu{}_{*\sigma}=A^\mu{}_{,\sigma}+A^{\rho*}\Gamma^\mu_{\rho\sigma}+aA^\mu\frac{\beta_{,\sigma}}{\beta}+bA^\mu\frac{\chi_{,\sigma}}{\chi} \tag{A.8}$$

$$A^\mu{}_{\sigma*\lambda}=A^\mu{}_{\sigma,\lambda}+A^\rho{}_\sigma{}^*\Gamma^\mu_{\rho\lambda}-A^\mu{}_\rho{}^*\Gamma^\rho_{\sigma\lambda}+aA^\mu_\sigma\frac{\beta_{,\lambda}}{\beta}+bA^\mu_\sigma\frac{\chi_{,\lambda}}{\chi} \tag{A.9}$$

where a and b are the β and χ powers of the tensor A in each case. Generalization to higher-rank tensors is straightforward. The star derivative as defined above does preserve units covariance as may be readily checked.

Just as in Riemannian geometry one finds that star derivatives do not commute. One defines the units covariant curvature tensor ${}^*R^\mu{}_{\lambda\gamma\sigma}$ to be

$$ {}^*R^\mu{}_{\lambda\gamma\sigma} \equiv {}^*\Gamma^\mu{}_{\lambda\gamma,\sigma} - {}^*\Gamma^\mu{}_{\lambda\sigma,\gamma} + {}^*\Gamma^\rho{}_{\lambda\gamma}{}^*\Gamma^\mu_{\rho\sigma} - {}^*\Gamma^\rho{}_{\lambda\sigma}{}^*\Gamma^\mu{}_{\rho\gamma} \tag{A.10} $$

$$ \begin{aligned} {}^*R^\mu{}_{\lambda\gamma\sigma} = {} & R^\mu{}_{\lambda\gamma\sigma} + (\delta^\mu_\gamma\delta^\rho_\lambda - g^{\mu\rho}g_{\gamma\lambda})(\ln\beta)_{;\rho\sigma} - (\delta^\mu_\sigma\delta^\rho_\lambda - g^{\mu\rho}g_{\sigma\lambda})(\ln\beta)_{;\rho\gamma} \\ & + \left(\delta^\mu_\sigma\frac{\beta_{,\gamma}}{\beta} - \delta^\mu_\gamma\frac{\beta_{,\sigma}}{\beta}\right)\frac{\beta_{,\lambda}}{\beta} + \left(g_{\lambda\gamma}\frac{\beta_{,\sigma}}{\beta} - g_{\lambda\sigma}\frac{\beta_{,\gamma}}{\beta}\right)g^{\mu\rho}\frac{\beta_{,\rho}}{\beta} \\ & + (\delta^\mu_\gamma g_{\sigma\lambda} - \delta^\mu_\sigma g_{\gamma\lambda})g^{\rho\alpha}\frac{\beta_{,\rho}}{\beta}\frac{\beta_{,\alpha}}{\beta} \end{aligned} \tag{A.11} $$

and finds

$$ A^\mu{}_{*\gamma\sigma} - A^\mu{}_{*\sigma\gamma} = {}^*R^\mu{}_{\lambda\gamma\sigma}A^\lambda \tag{A.12} $$

${}^*R^\mu{}_{\lambda\gamma\sigma}$ has the same index symmetries as $R^\mu{}_{\lambda\gamma\sigma}$. Notice that star derivatives do not commute even in flat space-time ($R^\mu{}_{\lambda\gamma\sigma} \equiv 0$) due to the β-dependent terms of (A.11). The associated contracted tensors formed from (A.11) are

$$ {}^*R_{\lambda\sigma} \equiv {}^*R^\mu{}_{\lambda\mu\sigma} = R_{\lambda\sigma} + 2\frac{\beta_{;\lambda\sigma}}{\beta} - 4\frac{\beta_{,\lambda}\beta_{,\sigma}}{\beta^2} + g_{\lambda\sigma}\left(\frac{\Box\beta}{\beta} + \frac{\beta_{,\rho}\beta_{,\mu}}{\beta^2}g^{\rho\mu}\right) \tag{A.13} $$

$$ {}^*R \equiv g^{\lambda\sigma}{}^*R_{\lambda\sigma} = R + 6\frac{\Box\beta}{\beta} \tag{A.14} $$

There is a unique second-rank tensor formed from (A.13) and (A.14) with the property that its star divergence vanishes

$$ \begin{aligned} {}^*G_{\lambda\sigma} \equiv {}^*R_{\lambda\sigma} - \tfrac{1}{2}g_{\lambda\sigma}{}^*R = {} & G_{\lambda\sigma} + 2\frac{\beta_{;\lambda\sigma}}{\beta} - 4\frac{\beta_{,\lambda}\beta_{,\sigma}}{\beta^2} \\ & - g_{\lambda\sigma}\left(2\frac{\Box\beta}{\beta} - \frac{\beta_{,\rho}\beta_{,\mu}}{\beta^2}g^{\rho\mu}\right) \end{aligned} \tag{A.15} $$

$$ {}^*G^{\lambda\sigma}{}_{*\sigma} \equiv 0 \tag{A.16} $$

which is the units covariant analog of the Einstein tensor. One also has the units covariant analog of the Bianchi identity

$$ {}^{*}R^{\mu}{}_{\lambda\gamma\sigma * \rho} + {}^{*}R^{\mu}{}_{\lambda\sigma\rho * \gamma} + {}^{*}R^{\mu}{}_{\lambda\rho\gamma * \sigma} \equiv 0 \tag{A.17} $$

The powers of various geometric quantities are

$$ \Pi_{\beta}(\beta) \equiv -1 \tag{A.18a} $$

$$ \Pi_{\beta}(g_{\lambda\sigma}) \equiv +2, \qquad \Pi_{\beta}(g^{\lambda\sigma}) = -2 \tag{A.18b} $$

$$ \Pi_{\beta}({}^{*}R^{\mu}{}_{\lambda\rho\sigma}) = 0, \qquad \Pi_{\beta}({}^{*}R_{\lambda\sigma}) = 0 \tag{A.19} $$

$$ \Pi_{\beta}({}^{*}R) = -2, \qquad \Pi_{\beta}(\det(g_{\lambda\sigma})) = +8 \tag{A.20} $$

$$ \Pi_{\beta}({}^{*}G_{\lambda\sigma}) = 0, \qquad \Pi_{\beta}({}^{*}G^{\lambda\sigma}) = -4 \tag{A.21} $$

with $\Pi_{\chi} = 0$ in each case. The β and χ powers of any pure number are always zero. These powers lead to the following important relations:

$$ \beta_{*\sigma} \equiv 0, \qquad [f(\beta)]_{*\sigma} \equiv 0 \tag{A.22} $$

$$ g_{\sigma\lambda * \rho} \equiv 0, \qquad g^{\sigma\lambda}{}_{*\rho} \equiv 0, \qquad \delta^{\sigma}{}_{\lambda * \rho} \equiv 0 \tag{A.23} $$

where $f(\beta)$ is an arbitrary continuous function of β and $\delta^{\sigma}_{\lambda}$ is the Kronecker delta.

Simplicity and LNH suggest that only one gauge variable is sufficient for our purposes, not two. Hence, define

$$ \chi(x) \equiv [\beta(x)]^{1-g} \tag{A.24} $$

where g is a constant. $\beta(x)$ is now the gauge variable which explicitly accounts for variable units standards. Instead of β power and χ power one has just *power* where if (A.4) holds then

$$ \Pi(A) = a + b(1-g) \equiv \alpha \tag{A.25} $$

Everything that was outlined above is unchanged except that β power equals power and χ power equals zero, i.e., in (A.7)–(A.9), $a = \alpha$ and $b = 0$.

The following relations will be found to be useful

$$A_{*\sigma} = \beta^{-\alpha}(\beta^{\alpha}A)_{,\sigma} \tag{A.26}$$

$$A^{\sigma}{}_{*\sigma} = \beta^{-\alpha-4}(\beta^{\alpha+4}A^{\sigma})_{;\sigma} \tag{A.27}$$

$$A^{\lambda\sigma}{}_{*\sigma} = \beta^{-\alpha-6}(\beta^{\alpha+6}A^{\lambda\sigma})_{;\sigma} - A^{\sigma}{}_{\sigma}g^{\lambda\rho}\beta_{,\rho}/\beta \qquad \text{if } A^{\lambda\sigma} = A^{\sigma\lambda} \tag{A.28}$$

$$A^{\lambda\sigma}{}_{*\sigma} = \beta^{-\alpha-4}(\beta^{\alpha+4}A^{\lambda\sigma})_{;\sigma} \qquad \text{if } A^{\lambda\sigma} = -A^{\sigma\lambda} \tag{A.29}$$

where α is the power of the tensor A in each case. Note that, for example,

$$\Pi(A^{\lambda}) = \Pi(g^{\lambda\sigma}A_{\sigma}) = \Pi(g^{\lambda\sigma}) + \Pi(A_{\sigma}) = \Pi(A_{\lambda}) - 2 \tag{A.30}$$

Finally, it is often useful to use a second fiducial standard. In this case β must be rescaled accordingly. In this series of papers I use the double vertical bar to denote units covariant derivatives where the fiducial standards are A units ($\beta \equiv \varphi$). Then $A^{\lambda}{}_{\|\sigma}$ is defined just like $A^{\lambda}{}_{*\sigma}$ except that in equations (A.1)–(A.29) above β is everywhere replaced by β/φ. Hence

$$\beta = 1 \Rightarrow A^{\lambda}{}_{*\sigma} = A^{\lambda}{}_{;\sigma} \tag{A.31}$$

$$\beta = \varphi \Rightarrow A^{\lambda}{}_{\|\sigma} = A^{\lambda}{}_{;\sigma} \tag{A.32}$$

and, for example, (A.27) becomes

$$A^{\sigma}{}_{\|\sigma} = (\beta/\varphi)^{-\alpha-4}\left[(\beta/\varphi)^{\alpha+4}A^{\sigma}\right]_{;\sigma} \tag{A.33}$$

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